LVII. On the Charge of Electricity carried by the Ions produced by Röntgen Rays.

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The following experiments were made in order to determine the magnitude of the charge of electricity carried by the ions which are produced when Röntgen rays pass through a gas.

The theory of the method used is as follows: By measuring the current passing through a gas exposed to Röntgen rays and acted upon by a known electromotive force, we determine the value of the product nev, when n is the number of ions in unit volume of the gas, e the charge on an ion, and v the mean velocity of the positive and negative ions under the electromotive force to which they are exposed.

Mr. Rutherford (Phil. Mag. vol. xliv. p. 422, 1897) has determined the value of v for a considerable number of gases; using these values, the measurement of the current through a gas gives us the product ne; hence if we can determine n, we can deduce the value of e.

The method I have employed to determine n is founded on the discovery made by Mr. C.T.R. Wilson (Phil. Trans. A, 1897, p. 265) that when Röntgen rays pass through dust-free air a cloud is produced by an expansion which is incapable of producing cloudy condensation when the gas is not exposed to these rays. When a determinate expansion is suddenly produced in dust-free air a definite and calculable amount of water is deposited in consequence of the lowering of the temperature of the air by adiabatic expansion. When the gas is exposed to the rays the ions caused by the rays seem to act as nuclei around which the water condenses. I have shown ('Applications of Dynamics to Physics and Chemistry,' p. 164) that on a charged sphere of less than a certain radius the effect of the charge in promoting condensation will more than counterbalance the effect of surface-tension in preventing it. So that a charged ion will produce a very small drop of water which may act as a nucleus. If each ion acts as the nucleus for a drop, then if we know the size of the drop and the mass of water deposited per unit volume, we shall be able to determine the number of drops, and hence the number of ions in unit volume of the gas. One part of the investigation is thus the determination of the size of the drops: this gives us n; and as we know from the electrical investigation ne, we have the means of determining e.

The measurement of the size of the drops in the cloud gave a great deal of trouble. Two methods were tried; at first I attempted to measure the size of the drops by an optical method; when a narrow beam of light from an arc lamp is sent through the cloud, and the light after passing through the cloud received on a screen, several coloured rings are visible. If we assume that these rings arise entirely from diffraction the size of the rings would enable us to deduce the size of the drops. The method, however, failed in practice from two causes. In the first place, in order to get the rays bright enough to allow their diameter to be accurately measured the fog must be dense, in order, however, to get a dense cloud the number of ions produced by the rays must be large; when, however, the number of ions is large experience shows that they are not all brought down by the first cloud formed by a sudden expansion. This is proved by the fact that if after the first cloud has subsided, the rays having been cut off immediately after the first expansion, another expansion be made, a second cloud will be formed, and though this is less dense than the first cloud it may require two or three expansions to remove the effects of previous exposure to the Röntgen rays. It is only when the ions are so few that no cloud is produced by the second expansion that we can feel any confidence that the number of drops in the first cloud is equal to the number of ions formed by the rays, and in this case the cloud is so thin that the coloured rays are not bright enough to allow their diameters to be accurately measured. Though this objection is fatal there is yet another reason against using this method of measuring the size of the drops, as observations made on the dimensions of the various coloured rings seemed to indicate that the rings are not produced entirely by diffraction, but that they are influenced by the interference of rays which have passed through the transparent drops with those which have not done so, and that therefore, we could not employ the usual formula connecting the size of the rings with the size of the drops.

The method finally employed to measure the size of the drops was to observe the rate at which the cloud sank and then to determine the radius of the drops from the formula

$$v = \frac{2}{9}\frac{ga^3}{\mu}$$

where v is the velocity with which the drops fall, a the radius of the drop, μ the coefficient of viscosity of the gas through which the drops fall, and g the value of gravity.

The velocity was determined by observing the time the top layer of the cloud, which was illuminated by an arc light, took to fall a given distance; observations made on the times taken to fall different stances showed that the rate of fall was uniform, so that the drops had reached their limiting velocity.

I began by making experiments to test whether the drops in the cloud formed by expansion were deposited round the ions which gave to the gas its electrical conductivity; this point is fundamental, as the method used in this paper to determine the charge carried by an ion depends on the assumption that it is the ionization of the gas which causes the fog produced by expansion, and that each ion can act as the nucleus for a water drop.

In the first place we have direct evidence of the power of an electrified particle to act as a nucleus for a drop of water, inasmuch as condensation takes place in a steam-jet when placed near an electrode from which electricity is escaping, and, further, Mr. Wilson has shown that a cloud is produced by expansion in dust-free air when an electrode discharging electricity is placed in the air. A more direct proof of the point under consideration is afforded by the following experiment: If the ions produced by the Röntgen rays act as nuclei for the drops, then, since these ions can be withdrawn from the gas by applying to it a strong electric field, it follows that a cloud ought not to be formed when the air which is expanded is exposed to a strong electric field while the rays are passing through it. This was found to be the case, and the experiment is a striking one. Two parallel plates were placed in the vessel containing the dust-free air; these plates were about 5 centim. apart, and were large enough to include the greater part of the air between them. The plates could be connected with the terminals of a battery of small storage-cells giving a potential-difference of about 400 volts. Röntgen rays passed through the gas between the plates: this gas had previously been freed from dust. When the plates were disconnected from the battery expansion produced a dense cloud; when, however, the plates were connected with the battery only a very light cloud was produced by the expansion, and this cloud was almost as dense when the Röntgen rays did not pass through the air as when they did.

Another point which had to be investigated was whether the cloud produced by the expansion caught all the ions. In this connexion it is necessary to point out that it is only possible to use expansions comprised within somewhat narrow limits. The ratio of the final to the initial volume of the gas has to be between 1.25 and 1.40. For, as Mr. Wilson (*loc. cit.*) has shown, when the expansion exceeds the larger of these values a dense cloud is produced even when the gas is not exposed to Röntgen rays, with these large expansions the cloud is so dense that the increase produced by the Röntgen rays is barely perceptible; while when the expansion is less than the smaller of these values no cloud at all is produced. With expansions comprised between these limits it was found that when the Röntgen rays were strong an increase in the strength of the rays did not increase the number of drops in the cloud, as determined by the rate of fall of the drops, nearly so fast as it increased the number of ions as measured by the electrical conductivity of the gas. But with these strong rays it was found that the effect of the Röntgen rays in producing a cloud was not exhausted by the first expansion, even when the rays were cut off immediately after that expansion took place; for a cloud was produced when a second expansion was made, and with strong rays it sometimes required six or seven expansions, occupying perhaps five or six minutes, before the effect of the rays had disappeared. In the face of this it is evident that when the rays are strong we are not entitled to assume that all the ions are brought down by the cloud produced by the first expansion. The first expansion, however, though it does not bring all the ions down, seems to increase the size of those left and makes them more permanent, for the ions which are left after the first expansion exert an appreciable cloud-producing effect for several minutes; whereas if no expansion had occurred the effect of the rays in producing a cloud would only have lasted for a few seconds after the rays had been cut off. Again, these modified ions are able to cause a cloud to settle with an expansion less than 1.25, the minimum expansion which gives a cloud with the original ions. When once a cloud has been produced the secondary clouds produced by subsequent expansions are but little affected by an electric field, this again indicating that the modified ions are larger and more sluggish than the original ones; the presence of these modified ions does not seem to give any appreciable conductivity to the gas. Mr. Wilson found that when in gas not exposed to Röntgen rays a dense cloud was produced in dust-free air by a large expansion and then allowed to settle, a subsequent small expansion (which under ordinary circumstances would not produce a cloud at all unless dust were present) would produce a cloud, and that it was necessary to produce several clouds and allow them to settle before the gas returned to its normal state. In this case Mr. Wilson's experiments seem to show that the original nuclei were excessively minute drops of water, and the formation of the subsequent cloud would seem to indicate that on those drops which did not grow large enough to be carried down by the first cloud some moisture was deposited, and that this was prevented from evaporating by some kind of chemical change at its surface such as the formation of hydrogen peroxide.

Whatever the explanation of these secondary clouds may be it is evident that when the rays are strong enough to produce them we cannot deduce the number of ions from observations on the primary cloud. In the experiments described below the intensity of the rays was weakened by interposing screens of aluminum between the bulb and the gas exposed to the rays until there was no more cloud produced by the second expansion than would have been produced if the gas had never been exposed to the rays.

Another point which had to be investigated was whether the expansion used was sufficient to bring down all the ions, or whether the number brought down increased with the amount of the expansion. To test this measurements were made of the rate of fall of the clouds formed under exposure to the rays by different expansions. The results of these experiments are shown in the following table:

Pressure of air 768.08 millim. Temperature 18°C.

	Time of fall through 25 millim.	
Expansion	with rays.	without rays.
$\frac{752.72}{535.72} = 1.4$	19	10
$\frac{752.72}{545.72} = 1.38$	18	6
$\frac{752.72}{555.72} = 1.35$	14	4

The amount of water deposited per cub. centim. by an expansion of 1.4 is 4.94×10^{-6} gram., while the amount deposited by an expansion of 1.35 is 4.74×10^{-6} gram. If N is the number of ions per cub. centim. in the first case when the rays are on, M the number when the rays are off, a the radius of the drops when the rays are on, b the radius when the rays are off, Q the quantity of water deposited:

$$\mathrm{N}\tfrac{4}{3}\pi a^3 = \mathrm{M}\tfrac{4}{3}\pi b^3 = \mathrm{Q}.$$

The rate of fall varies as the square of the radius of the drops, so that

$$\frac{a}{b} = \frac{\sqrt{10}}{\sqrt{19}}$$

If dashed letters refer to the second expansion,

$$\mathrm{N}'\tfrac{4}{3}\pi a'^3 = \mathrm{M}'\tfrac{4}{3}\pi b'^3 = \mathrm{Q}',$$

so that

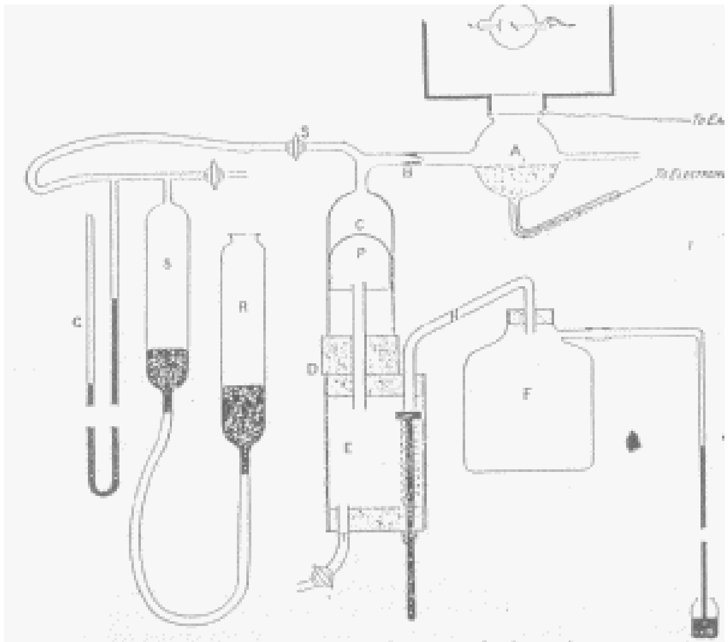
$$\begin{aligned}\frac{\mathrm{N}-\mathrm{M}}{\mathrm{N}'-\mathrm{M}'} &= \frac{\mathrm{Q}\left\{\frac{1}{a^3}-\frac{1}{b^3}\right\}}{\mathrm{Q}'\left\{\frac{1}{a'^3}-\frac{1}{b'^3}\right\}} \\ &= \frac{4.94}{4.74}\frac{\{19\sqrt{19}-10\sqrt{10}\}}{\{14\sqrt{14}-4\sqrt{4}\}} \\ &= 1.2 \text{ approximately.}\end{aligned}$$

Thus the number of the ions produced by the rays which are caught by the larger expansion is slightly greater than that caught by the former. I think that the greater rapidity with which the larger expansions are made, in consequence of the greater time the driving force acts on the piston whose motion produces the expansion, is sufficient to account for this; for when the expansion is slow the drops first formed can grow before the expansion is completed, and thus rob the others of the water-vapour, so that we should expect to get slightly more drops as we increased the rapidity of the expansion.

Some experiments made with smaller expansions seemed rather to indicate a considerable increase in the number of ions deposited when the expansion was taken from below 1.3 to above it. An increase which seemed rather too large to be attributed wholly to the increased velocity of expansion, and to suggest that the ions had not all the same power of acting as nuclei. I hope to make an independent investigation of this point, as it is evidently one which might have considerable bearing on the problems of atmospheric electricity; for if the negative ions, say, were to differ in their power of condensing water around them from the positive, then we might get a cloud formed round one set of ions and not round the other. The ions in the cloud would fall under gravity, and thus we might have separation of positive and negative ions and the production of an electric field, the work required for the production of the field being done by gravity.

To return, however, to the experiments under consideration. The method employed for making the cloud and for measuring the expansion is the same as that used by Mr. Wilson and described by him in the 'Proceedings of the Cambridge Philosophical Society,' vol. ix. p. 333. The gas which is exposed to the rays is contained in the vessel A; this vessel communicates by the tube B with the vertical tube C, the lower end of this tube is carefully ground so as to be in a plane perpendicular to its axis, and is fastened down to the indiarubber stopper D. Inside this tube there is an inverted thin-walled test-tube, P, with the lip removed and the open end ground so as to be in a plane perpendicular to the axis of the tube. The test-tube slides freely up and down the larger tube and serves as a piston. Its lower end is always below the surface of the water which fills the lower part of the outer tube. A glass tube passing through the indiarubber stopper puts the inside of the test-tube in connexion with the space E. This space is in connexion with an exhausted vessel, F, through the tube H. The end of this tube is ground flat and is closed by an indiarubber stopper which presses against it; the stopper is fixed to a rod, by pulling the rod down smartly the pressure inside the test-tube is lowered and it falls rapidly until the test-tube P strikes against the indiarubber stopper. The tube T, which can be closed by a stop-cock, puts the vessel E in connexion with the outside air. The tubes R and S are for the purpose of regulating the amount of the expansion. To do this, the mercury-vessel R is raised or lowered when the test-tube is in the lowest position until the gauge G indicates that the pressure in A is the desired amount below the atmospheric pressure. The clip S is then closed, and air is admitted into the interior of the piston by opening the clip T. The piston then rises until the pressure in A differs from the atmospheric pressure only by the amount required to support the piston, this is only a fraction of a millimetre.

If Π is the barometric pressure, then the pressure of the air before expan-



sion is

$$P_1 = \Pi - \pi,$$

where π is the maximum vapour-pressure of water at the temperature of the experiment. The pressure of the air after the expansion when the temperature has risen to its former value is

$$P_2 = P_1 - p,$$

where p is the pressure due to the difference of level of the mercury in the two arms of the gauge.

Thus if v_2 is the final and v_1 the initial volume,

$$\frac{v_2}{v_1} = \frac{P_1}{P_2} = \frac{\Pi - \pi}{\Pi - \pi - p}$$

A is the vessel in which the rate of fall of the fog was measured and the electrical conductivity of the gas tested. It is a glass tube about 36 millim. in diameter covered with an aluminum plate; a piece of wet blotting-paper is placed on the lower side of the plate and the current of electricity passed from the blotting-paper to the horizontal surface of the water in this vessel. The blotting-paper was placed over the aluminum plate to avoid the abnormal ionization which occurs near the surface of a metal against which Röntgen rays strike normally. M. Langevin has shown that this abnormal ionization is practically absent when the surfaces are wet.

The coil and focus-bulb producing the rays were placed in a large iron tank elevated on supports; in the bottom of the tank a hole was cut and closed by an aluminum window. The vessel A was placed underneath this window and the bulb giving out the rays some distance behind it, so that the beam of rays escaping from the tank were not very divergent. The rays were reduced in intensity to any required degree by inserting different numbers of layers of tinfoil or sheets of aluminum between the bulb and the vessel. The tank and the aluminum plate at the top of A were connected with earth and with one pair of quadrants of an electrometer. The other pair of quadrants were connected with the water-surface B; this surface was charged up by connecting it with one of the poles of a battery consisting generally of two Leclanché cells, the other pole of which was connected with earth. After the surface was charged it was disconnected from the battery and the insulation of the apparatus tested by observing whether there was any leak when the Röntgen rays were not on: the insulation having been proved to be good, the rays were turned on, when the charge began to leak; by measuring the rate of leak, the quantity of electricity crossing in one second the gas exposed to the rays can be determined if the capacity of the system is known. The effective capacity of the system consisting of the discharging vessel, the connecting wires, and the quadrants of the electrometer depends to a large extent on the charge in the electrometer, and increases so quickly with the charge that the rate of movement of the spot of light reflected from the mirror of the electrometer increases but slowly when the charge in the electrometer is increased beyond a certain value. The reason for this is shown by the following investigation.

Let Q_1 be the charge on the system consisting of the pair of quadrants and the apparatus connected with it, V_1 the potential of this pair of quadrants, V_2 the potential of the other pair, and V_3 the potential of the needle; then we have

$$\mathrm{Q}_1 = q_{11}\mathrm{V}_1 + q_{12}\mathrm{V}_2 + q_{13}\mathrm{V}_3,$$

where q_{11}, q_{12}, q_{13} are coefficients of capacity. Let θ be the azimuth of the needle, then if the two pairs of quadrants and the needle are at the same potential, Q_1 will not depend upon θ if the quadrants are symmetrical with respect to the axis of the needle. Hence

$$\frac{dq_{11}}{d\theta} + \frac{dq_{12}}{d\theta} + \frac{dq_{13}}{d\theta} = 0.$$

If the needle is initially placed symmetrically with respect to the quadrants, then

$$\frac{dq_{12}}{d\theta} = 0$$

approximately when θ is small.

Thus if q_{11}, q_{13} denote the values of q_{11}, q_{13} when θ is zero we have approximately, if $\beta = \frac{dq_{13}}{d\theta}$,

$$q_{11} = q_{11} - \beta\theta \; ; \qquad\qquad q_{13} = \mathrm{q}_{13} + \beta\theta,$$

and

$$\mathrm{Q}_1 = \mathrm{q}_{11}\mathrm{V}_1 + \mathrm{q}_{12}\mathrm{V}_2 + \mathrm{q}_{13}\mathrm{V}_3 + \beta\theta(\mathrm{V}_3 - \mathrm{V}_1) \; ;$$

if $V_2 = 0$ we have, since the deflexion of the needle is approximately proportional to the product of the potential-difference between the quadrants and the potential of the needle,

$$\theta = k\mathrm{V}_1\mathrm{V}_3.$$

Hence $Q_1 = q_{11}V_1 + q_{13}V_3 + k\beta V_1V_3^2 - K\beta V_1^2V_3$;
the fourth term on the right-hand side is small compared with the third; hence we have

$$\frac{d\mathrm{Q}_1}{d\mathrm{V}_1} = \mathrm{q}_{11} + k\beta\mathrm{V}_3^2.$$

Thus the effective capacity is $q_{11} + k\beta V_3^2$.

The effective capacity was measured by connecting a parallel-plate condenser with the quadrants and then observing, when the system was insulated, the change in the deflexion when the distance between the plates was increased by a known amount. Supposing the capacity of the parallel-plate condenser was C in the first position and C' in the second, then we have, if V_1 and V_1' are the corresponding potentials,

$$\begin{aligned} \mathrm{Q}_1 &= (\mathrm{q}_{11} + \mathrm{C})\mathrm{V}_1 + \mathrm{q}_{13}\mathrm{V}_3 + \beta k\mathrm{V}_1\mathrm{V}_3^2 \\ &= (\mathrm{q}_{11} + \mathrm{C}')\mathrm{V}_1' + \mathrm{q}_{13}\mathrm{V}_3 + \beta k\mathrm{V}_1'\mathrm{V}_3^2; \end{aligned}$$

thus

$$\frac{\mathrm{V}_1}{\mathrm{V}_1'} = \frac{\mathrm{q}_{11} + \beta k \mathrm{V}_3^2 + C'}{\mathrm{q}_{11} + \beta k \mathrm{V}_3^2 + C}.$$

Since V_1, V_1' are proportional to the deflexion in the two cases and C′ and C are known, this equation enables us to calculate $q_{11} + \beta k \mathrm{V}_3^2$, the effective capacity of the system.

If, when the rays are on, the movement of the spot of light indicates a change in the potential equal to V per second, then the quantity of electricity flowing in that time across the cross-section of the vessel exposed to the rays is CV. But if n is the number of ions, both positive and negative, per cubic centimetre of the gas, u_0 the mean velocity of the positive and negative ions under unit potential gradient, A the area of the plates, E the potential-gradient, this quantity of electricity is also equal to neu_0 EA, hence we have

$$\mathrm{CV} = neu_0 \mathrm{EA} \;;$$

so that if we know n and u_0 we can from this equation deduce the value of e.

The method of making the experiments was as follows:—The aluminum plate and the water-surface were connected with the poles of two Leclanché cells, and the rate of fall, r_1, of the drops produced by an expansion when the rays were not on measured; the rays were now turned on, and the rate of fall, r_2, of the cloud now produced by the expansion determined; the rays were now turned off, and a third expansion taken, and the rate of fall of the cloud, r_3, found; if r_3 was appreciably less than r_1, it was taken as indicating that the ions produced by the rays were too numerous to be caught by one expansion, and the intensity of the rays was therefore cut down by inserting aluminum foil between the bulb and the vessel; this process was repeated until r_3 was equal to r_1, and then it was assumed that all the ions were caught by the cloud produced by the expansion. From the rate of fall the size of the drops was calculated from the formula

$$v = \frac{2}{9}\frac{ga^2}{\mu},$$

where v is the velocity, a the radius of the drop, and μ the coefficient of viscosity of the gas through which the drop falls. If q is the mass of water deposited from a cubic centimetre of the gas, we have

$$q = n\frac{3}{4}\pi a^3.$$

The method used to determine q is that given by Wilson in his paper on the formation of clouds in dust-free air (Phil. Trans. 1897, A, p. 299). We have the equation

$$\mathrm{L}q = \mathrm{CM}(t - t_2),$$

where L is the latent heat of evaporation of water, C the specific heat of the gas at constant volume, M the mass of unit volume of the gas, t_2 the lowest temperature reached by the expansion, t the temperature when the drops are fully grown.

Since

$$q = \rho_1 - \rho,$$

where ρ_1 is the density of the water-vapour before condensation begins, and ρ the density at the temperature t; hence we have

$$\rho = \rho_1 - \frac{\mathrm{CM}}{\mathrm{L}}(t - t_2).$$

Since ρ is a function of t, this equation enables us to determine t. If x is the ratio of the final to the initial volume and t_0 the temperature before expansion, then, since the mass of unit volume of air is .00129 grm. at 0° C. and under a pressure of 760 mm. of mercury, we have

$$\mathrm{M} = \frac{.00129}{x} \times \frac{273}{273 + t_0},$$

if we take the initial pressure to be 760.

Again,

$$\rho_1 = \frac{\rho_0}{x},$$

where ρ_0 is the density of water-vapour at the temperature t_0. The cooling caused by the expansion is determine by the equation

$$\log \frac{273 + t_0}{273 + t_2} = .41 \ \log x;$$

$$\mathrm{C} = .167; \qquad \mathrm{L} = 606.$$

Thus

$$\rho = \frac{\rho_0}{x} - \frac{.167}{606} \times \frac{.00129}{x} \frac{273}{273 + t_0}(t - t_2).$$

Let us apply these equations to a special case. In one of the experiments $t_0 = 16°$C. and

$$x = \frac{760 - 13.5}{760 - 13.5 - 197} = 1.36,$$

$$\log \frac{273 + 16}{273 + t_2} = .41 \ \log 1.36$$
$$= \log 1.134 \ ;$$

hence

$$\begin{aligned} 273 + t_2 &= 254.8, \\ t_2 &= -18.2, \\ \rho_0 &= .0000134, \\ \frac{\rho_0}{1.36} &= 98.4 \times 10^{-7} \end{aligned}$$

and

$$\frac{.167 \times .00129 \times 273}{606 \times 1.36 \times 289} = 2.46 \times 10^{-7}$$

hence

$$\rho = 98.4 \times 10^{-7} - 2.46 \times 10^{-7}(t + 18.2).$$

If we put $t = 1.2$, we get from this equation

$$\rho = 50.7 \times 10^{-7},$$

which is very nearly the value of ρ at 1.2°C.; hence we conclude $t = 1.2$ and q, the amount of water deposited per unit volume of the expanded gas, is 47.7×10^{-7} grms.

It was found that, when the rays were on, the velocity of the drops was .14 cm./sec., while without the rays the velocity was .41 cm./sec.

Connextion between the Velocity and Size of the Drop.

If v is the velocity with which a drop of water of radius a falls through a gas whose coefficient of viscosity is μ, then if we neglect the density of the gas in comparison with that of the drop

$$\tfrac{4}{3}\pi g a^3 = 6\pi\mu a \mathrm{V}\frac{1 + 4\frac{\mu}{\beta a} + 6\left(\frac{\mu}{\beta a}\right)^2}{\left(1 + \frac{3\mu}{\beta a}\right)^2}$$

(see Lamb's 'Hydrodynamics,' ed. i. p. 230), where β is the slipping coefficient. If there is no slip between the sphere and the gas, β is infinite, and we have

$$\mathrm{V} = \frac{2}{9}\frac{ga^2}{\mu}; \ldots\ldots\ldots\ldots \tag{1}$$

while if $\mu/\beta a$ is large we have

$$\mathrm{V} = \frac{1}{3}\frac{ga^2}{\mu}.$$

Since a occurs in the denominator in the terms involving $1/\beta$, the influence of slipping on the motion of very small spheres such as those we are considering will be much more important than its influence on the motion of spheres of the size used for the bobs of pendulums, for which the influence of slipping has been shown to be too small to be detected. We cannot, therefore, without further consideration neglect in our case the terms involving $1/\beta a$. Some light is thrown on the question by the Kinetic Theory of Gases, for according to that theory (see Maxwell, "Stresses in Rarefied Gases;" Collected Works, vol. ii. p. 709) μ/β is of the order of the mean free path of a molecule, *i.e.*, for air at atmospheric pressure of the order 10^{-5} centim.; hence, if a is large compared with the mean free path, we should expect the relation between the velocity and size to be that given by equation (1).

Taking the equation

$$v = \frac{2}{9}\frac{ga^2}{\mu}.$$

and putting

$$v = .14, \qquad g = 981, \qquad \mu = 1.8 \times 10^{-4},$$

we find

$$\begin{aligned} a^2 &= 11.5 \times 10^{-8}, \\ a &= 3.39 \times 10^{-4}, \\ \tfrac{4}{3}\pi a^3 &= 1.63 \times 10^{-10}. \end{aligned}$$

As the radius of the drop is considerable compared with the mean free path in air at atmospheric pressure we may feel some confidence that equation (1) will be true for drops of this size.

Hence

$$n = \frac{q}{\frac{4}{3}\pi a^3} = 2.94 \times 10^4$$

This is the number of ions in 1 cub. centim. of the expanded gas; the number in 1 cub. centim. of the gas before expansion

$$= 2.94 \times 1.36 \times 10^4 = 4 \times 10^4.$$

We now consider the electrical part of the experiment. The electrometer gave a deflexion of 90 scale-divisions for two Leclanché cells, the capacity of the system consisting of the cell containing the gas exposed to the rays, the connecting wires, and the quadrants was 38, on the electrostatic system of units. The diameter of the circular electrodes between which the leak took place was 3.6 centim., and the distance between them 2 centim. When the rays were on, and the potential-difference between the electrodes that due to two Leclanchés, the leak was at the rate of 9 scale-divisions per minute; hence if E is the electromotive force of a Leclanché cell, the quantity of electricity passing in one second through a cross-section of the discharge-tube is equal to

$$\frac{38}{300}\mathrm{E}$$

But this is equal to

$$\mathrm{A}neu_0\mathrm{E}',$$

where A is the area of the electrodes and equals $\pi(1.8)^2$, n the number of ions per cub. centim. $= 4 \times 10^4$, e the charge on an ion, u_0 the mean velocity of the positive and negative ion under unit potential gradient, Mr. Rutherford found this to be $1.6 \times 3 \times 10^2$. E' is the potential gradient, assumed to be uniform, in our case it was E. Substituting these values, we get

$$\frac{38}{300}\mathrm{E} = \pi(1.8)^2 \times 4 \times 10^4 \times e \times 4.8 \times 10^2 \times \mathrm{E};$$

hence

$$e = 6.3 \times 10^{-10}.$$

In the preceding investigation we have assumed that the nuclei producing the cloud are those which cause the conductivity, and are produced by the rays; there is, however, a small cloud produced even when no rays are on; if we assume that the nuclei which produce this cloud are still active when the rays are on, it follows that in the preceding investigation we have overestimated the number of ions engaged in carrying the current by the number of nuclei present when the rays are not passing through the gas. As the cloud fell three times faster when the rays were not on than it did when the rays were on, the number of nuclei when the rays are not on is to the number when the rays are on as 1 is to $3^{\frac{3}{2}}$, or as 1: 5.2; hence 1/5.2 of the nuclei are not engaged in carrying the current, so that to get the charge on the ions we must increase the value just given in the ratio of $1 + 1/5.2$ to 1; this makes

$$e = 7.4 \times 10^{-10}.$$

The results of other experiments on air are given in the following table:—

Expansion	Temp.	Current through gas.	Rate of fall of cloud.	e uncorrected for nuclei present without rays.	e corrected.
1.36	16	.243 E	.09	6.7×10^{-10}	7.6
1.36	16	.133 E	.147	6.4	7.2
1.38	16	.143 E	.156	7.3	8.4
1.36	16	.196 E	.104	6.3	7.4
1.36	16	.115 E	.125	5.0	6.0

The mean of these values and the one previously obtained is

$$e = 7.3 \times 10^{-10} \text{ electrostatic units.}$$

Another correction has to be made to allow for the conductivity of the walls of the vessel A due to the film of moisture with which it is coated. Though the walls are insulated from the aluminum plate at the top of the vessel, and there is no leak between them when the rays are not passing through the glass, the conductivity of the glass when the rays are on causes the current to travel partly from the aluminum plate and along the walls of the vessel instead of wholly through the air as has been assumed in the calculations. To estimate the correction two vessels were made of the same shape and size, the one precisely similar to that used in these experiments with water at the bottom, while the other had the walls covered with shellac varnish, and the water at the bottom was replaced by an aluminum plate of the same area, and at the same distance from the top plate as the upper surface of the water in the other vessel. The aluminum covers for the two vessels were cut from the same sheet of metal. When these vessels were exposed to the Röntgen rays the current through the vessel containing water was to that through the other vessel as 9 to 8. Thus the current passing directly between the plates was 8/9 of the current observed. Applying this correction the mean value of e is equal to

$$\tfrac{8}{9} \times 7.3 \times 10^{-10} = 6.5 \times 10^{-10}.$$

A series of experiments of a similar kind were made, using hydrogen instead of air. The number of ions in this gas was smaller than in the case of air, and the smaller viscosity of hydrogen made the drops fall much faster; the drops formed without the rays fell so fast, only taking a second or two, that the rate could not be determined with accuracy, nor was it certain that they had reached a steady state. The velocity of the hydrogen ion through hydrogen under unit potential gradient is taken as three times that of air and the coefficient of viscosity as 9.3×10^{-5}. The results of the experiments are given in the following table:—

Expansion	Temp.	Current through gas.	Rate of fall of cloud.	e uncorrected for nuclei present without rays.
1.36	16	0.21	.415 cm/sec	6.3×10^{-10}
1.38	16	0.127	.5	5.5
1.37	17	0.083	.83	6.9
1.35	17	0.19	.5	8.0
	Mean			6.7×10^{-10}

The value of e for hydrogen has not been corrected in the way that the value of e for air has been by allowing for the part of the cloud formed independently of the rays. Allowing for this the experiments seem to show that the charge on the ion in hydrogen is the same as in air. This result has very evident bearings on the theory of the ionization of gases produced by the Röntgen rays.

In obtaining the above values certain assumptions have been made to simplify the calculation which would have the effect of making the value of e differ from the true value. Thus, for example, we have assumed that the potential gradient is constant between the plates. Prof. Zeleny has shown (Phil. Mag. July 1898) that this is not strictly true; the potential fall near the plates is greater than the average, while that in the body of the gas is less. Thus the potential gradient in the gas is less than the difference of potential between the plates divided by the distance between them, which is the value we took in the preceding calculations. For the very much enfeebled rays we used in these experiments the difference between the true and the assumed value is so small that it did not seem worth while making the elaborate experiments necessary to calculate the correction, especially as the variations in the coil &c. produced disturbing effects far greater than would result from this cause. We have assumed, too, that all the ions produced by the rays are brought down by the cloud; if there were any left behind then the value we have deduced for the charge would be greater than the true value. The value we have found for the charge on the ion produced by Röntgen rays is greater than that usually given for the charge on the hydrogen atom in electrolysis. There seems, however, to be no valid reason against the latter charge having as high a value as that we have found. We get from the laws of electrolysis, if e is the charge on the hydrogen ion in electrostatic units, N the number of molecules in 1 cub. centim. at standard temperature and pressure,

$$\mathrm{N}e = 129 \times 10^8$$

(see Richarz, *Bonn Sitzungsherichten*, 1891, p. 23); if we take $e = 6.5 \times 10^{-10}$ we get

$$\mathrm{N} = 20 \times 10^{18},$$

where N, deduced from experiments on the viscosity of air, is 21×10^{18}. Though the measurements of the coefficients of viscosity of other gases give in general higher values of N, yet the agreement between the value of N deduced from these experiments and the value of N got by the Kinetic Theory of Gases by viscosity experiments is sufficient to show that that theory is consistent with the value we have found for e being equal to, or at any rate of the same order as, the charge carried by the hydrogen ion in electrolysis.

In connexion with this result it is interesting to find that Professor H. A. Lorentz (*Koninkligke Akademie van Wetenschappen te Amsterdam*, April 6, 1898) has shown that the charge on the ions whose motion causes those lines in the spectrum which are affected by the Zeeman effect is of the same order as the charge on the hydrogen ion in electrolysis.

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